Geometric Presicion

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Introduction

Geometry is the branch of mathematics that study of elements of space such as points, lines, angles, shapes and solids, their measurements and the relationships between them. The term *geometry,* meaning *measuring earth*, is derived from the Greek words *geo,* meaning *earth,* and *metron,* meaning *measure.*

Geometry is so fundamental that 25 centuries ago the Greeks used it to form the foundation of all of mathematics. In addition of studying geometry for its own sake, we will, like the Greeks, use it as means to help us understand many other mathematical subjects such as numbers, counting and computational operations. We can demonstrate, indeed we can derive — in fact, this is exactly what the Creeks — the complete number system graphically.

Points and Lines

The most fundamental elements of geometry are the *point* and the *line*.

Definitions

- ***Point.*** *A point is a location in space. A point has no dimensions.* That is, a point has no size.

- ***Line.*** *A line is a continuous collection of points.* That is, on a line, between any two points there exists at least another point that is different from either one of the given two points.

  It is common to use the term *line* with respect to a *straight line*.

  - ***Straight line.*** *A straight line is the shortest line that lies between two points.* A line has only one dimension — its ***length***. A line has no width.
  - ***Curve.*** *A curve is any line that lies between two points that is not a straight line.*

  From this point on, when we use the term *line* we mean *straight line* unless we specifically indicate otherwise. We will use the term *curve* to refer to a line that is not straight.

  A line can be continued at either or both of its end indefinitely.

  When the subject of our interest is a section of a line that is bound by two points we think of it as and refer to a *line segment*. However, we often abbreviate it and use the word *line.* However, we can <u>name</u> the line by using the names of the two points that terminate it.

- ***Plane.*** *A plane is a continuous collection of lines.* That is, on a plane, between any two lines there exists at least another line that is different from either one of the given two lines. Since the term *line* means to *straight line*, the plane is flat. (A curved plane is formed by a continuous collection of curves.) A plane has two dimensions — its length and its width. The plane has no thickness or height.

  When the subject of our interest is a section of a plane that is bound by lines we refer to it as a *surface.* At least three lines are necessary to bound a surface.

Drawing Points and Lines

In order to study various geometric objects we need to visualize them. More over we must be able to communicate with each other and even with ourselves, as we recall thoughts that we had in the past, effectively. Therefore, images that we construct in our mind's eye of geometric objects are not enough. We must construct them graphically so we can communicate our ideas with others and recall them days and years later.

Unfortunately, The best we can draw is limited by the physical properties of our tools. So we agree that our drawings are nothing but representations of the ideal figures that mathematics and geometry require. Still we strive to be as close as possible to the precise notion we have in mind. So, when we want to draw a straight line we use a straight edge to guide our drawing implement.

We also recognize that the closest we can get to having no dimension is limited by the fineness of the tip of our drawing device. When we draw with a pencil the line cannot be thinner than the sharpest point we can get at the tip of the pencil. So, as long as we want to see lines, lines always must have some thickness. But we refine the line by using a fine drawing implement.

What about drawing a point? After all, a point should have no dimension at all.

It is common practice by many textbooks to use **black dots** to represent points in drawings. Dots are not points. They are small circles. Actually, dots are large enough so several different lines, having a thickness that is equal to the that of the already drawn line(s), can be drawn through the same dot. This can cause serious problems when students measure distances with a compass and try to construct congruent objects through points that are identified by dots or elsewhere.

Consider for example the following drawing.

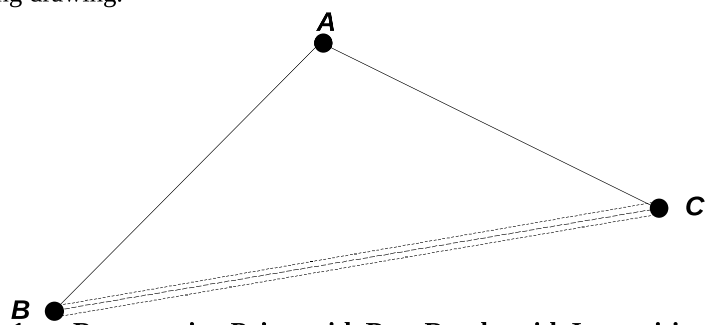

Figure 1. Representing Points with Dots Results with Imprecision. Given 3 points A, B and C and the lines AB and AC. To construct the triangle ABC, draw the line BC. But as the image shows, at least three lines are possible. Which is the correct one? Is it any of the drawn lines or is it a different one? How do you decide?

But geometry gives as the optimal means to define a point. It is based on the following duality. These two principals are fundamental to geometry.

2 Points Define a Line

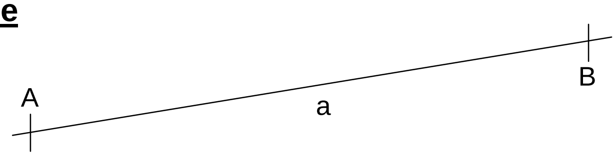

Figure 2. 2 Points Define a Line

2 Lines Define a Point

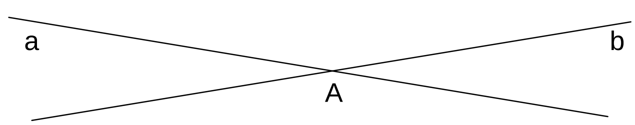

Figure 3. 2 Lines Define a Point

Back To The Drawing Example

To mark a point as precisely as we can we use the intersection of two lines. If we draw two intersecting lines, they may extend slightly beyond the intersection so the point is easily identifiable. In the drawing below (Figure 4) this was done to mark point A. If we need to mark a point on a given line, we use short line segments, called ***tick marks***. This was done in Figure 4 to mark points B and C. Then the line BC can be drawn. There is no need to erase the tick marks. (Using tick marks is a common practice of professional draftsmen, architects, and so on.)

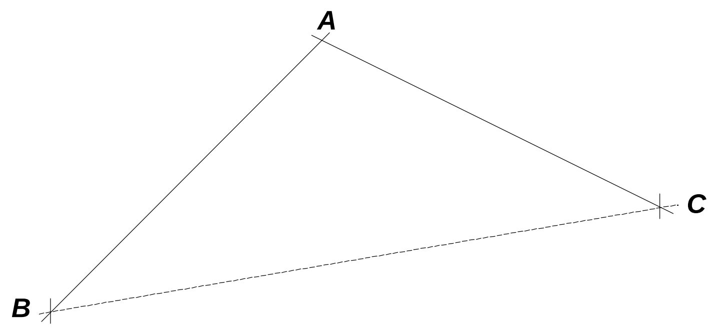

Figure 4. Points Are Constructed by Simple Line Intersections.
Lines are extended beyond the desired point to make it more visible (See point A). Short line segments, called, ***tick marks***, were drawn to mark points on a given line as was done for points B and C.
Isn't this method of being as precise as possible better?